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ABSTRACT

An approach to simultaneous optimization of assignments of subjects to treatments followed by an end-of-mastery test is presented using the framework of Bayesian decision theory. Focus is on demonstrating how rules for the simultaneous optimization of sequences of decisions can be found. The main advantages of the simultaneous approach, compared to the separate approach, are the more efficient use of data and the fact that more realistic utility structures can be used. The utility structure dealt with in this combined decision problem is a linear utility function. Decision rules are derived for quota-free as well as quota-restricted assignment situations when several culturally biased subpopulations of subjects are to be distinguished. The procedures are demonstrated with an empirical example of instructional decision making in an individualized study system that involves combining two elementary decisions. A 34-item list of references, three data tables, and one figure are included. (Author/TJH)

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Research Report
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A Simultaneous Approach to Optimizing Treatment Assignments with Mastery Scores

Hans J. Vos

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Abstract

An approach to simultaneously optimize assignments of subjects to treatments followed by an end-of-mastery test is presented using the framework of Bayesian decision theory. The main advantages of the simultaneous approach compared to the separate are the more efficient use of data and the fact that more realistic utility structures can be used. The utility structure dealt with in this combined decision problem is a linear utility function. Decision rules are derived for quota-free as well as quota-restricted assignment situations when several culturally biased subpopulations of subjects are to be distinguished. The procedures are demonstrated with an empirical example of instructional decision making in an individualized study system.

Keywords: decision theory, culture-fair placement, linear utility, quota-free placement, quota-restricted placement.

Introduction

Statistical decision problems arise when a decision maker is faced with the need to choose a preferred action that is optimal in some sense. Moreover, one decision problem often leads to another, which, in turn, leads to a next one, and so on. An example is test-based decision making in an individualized study system (ISS), which can be conceived of as an instructional network consisting of various types of decisions as nodes (Vos & van der Linden, 1987; Vos, 1990). How should we model and analyse such sequences of decision problems within a Bayesian decision-theoretic approach (e.g. DeGroot, 1970; Ferguson, 1967; Keeney & Raiffa, 1976; Lindgren, 1976)? In general, two main approaches can be distinguished: either each decision can be optimized separately or all decisions simultaneously. In the former approach, the expected utility of each separate decision is maximized sequentially while in the latter the overall expected utility of all decisions is maximized simultaneously.

It is the purpose of this paper to demonstrate how rules for the simultaneous optimization of sequences of decisions can be found. Compared with the separate optimization of decisions, two main advantages can be identified. First, when optimizing rules, decisions to be made later in the decision network can already be taken into account. As a result of this approach, rules can be found that make more efficient use of the data in the decision network. Also, the overall

expected utility will be increased. Second, more realistic utility structures can be handled by the simultaneous approach.

Van der Linden (1985, 1988) has given an elegant typology of decision problems in educational and psychological testing. Each decision problem from this typology can be viewed as a specific configuration of three basic elements, namely a test, a treatment, and a criterion. With the aid of these elements, the following four different types of decision problems can be identified: selection, mastery, placement, and classification.

Well-known examples of the four types of decision making in the field of education are admission of students to educational programs (selection), pass-fail decisions (mastery), the aptitude-treatment-interaction paradigm in instructional psychology where students are allowed to reach the same educational objectives via different instructional treatments (placement), and vocational-guidance situations where, for instance, most appropriate continuation-schools must be identified (classification).

Each of the four elementary decision problems can be formalized as a problem of (empirical) Bayesian decision making. In Hambleton and Novick (1973), Huynh (1976, 1977), Mellenbergh & van der Linden (1981), Novick and Petersen (1976), Petersen (1976), Petersen and Novick (1976), van der Linden (1980, 1981, 1987), and Vos (1988), it is indicated how optimal decision rules can be found for these problems (analytically or numerically).

In the present paper, the emphasis is on deriving simultaneous optimal decision rules for combinations of the elementary decisions. To illustrate the approach, a placement and a mastery decision will be combined into a simple decision network (see also Figure 1). The difference between the separate and the simultaneous approach can be demonstrated by the combined placement-mastery decision of Figure 1. In the separate approach, first optimal placement rules are found by maximizing the overall expected utility for the separate placement decision (e.g., van der Linden, 1981). Next, optimal mastery rules are found by maximizing the overall expected utility for the separate mastery decision (e.g., Hambleton & Novick, 1973). In the simultaneous approach, however, the optimal placement as well as the optimal mastery rules are found by maximizing the overall expected utility of both decisions simultaneously. Also, when optimizing treatment-assignment rules, pass-fail decisions to be made later can already be taken into account; hence, the first advantage of the simultaneous approach is nicely demonstrated by Figure 1.

Besides the pure forms and combinations with each other, one or more of the following generalizations may apply (van der Linden, 1988) to the elementary decisions:

(1). Multiple populations. The presence of different subpopulations reacting differently to the test items may create the problem of culture-fair decision making.

(2). Quota restrictions. Due to shortage of resources, the number of vacant places in some treatments is

restricted.

(3). Multivariate test data. The decisions are based on data from multiple tests.

(4). Multivariate criteria. The success of a treatment has to be measured on more than one criterion each reflecting a different aspect of the treatment.

In the present paper, only restrictions with respect to the presence of subpopulations and the number of vacant places in some treatments will be assumed. First, we elaborate the decision-theoretic aspects of culture-fair decision making for a quota-free placement-mastery problem. For a linear utility function the decision rule that optimizes simultaneously the treatment assignments and the pass-fail decisions to be taken after the treatments are given. Next, optimal rules will be derived if allocation quota considerations have to be taken into account. Finally, optimal cutting scores for quota-free as well as quota-restricted combined decisions will be presented for an empirical application to instructional decision making in an ISS. In the numerical example, it is further assumed that the students can be separated into two subpopulations referred to as the disadvantaged and the advantaged populations.

With respect to the applicability of the approach presented in this paper, the following should be regarded. Although the area of individualized instruction is a useful application of simultaneous decision making, it should be emphasized that the procedures advocated in this paper have a

larger scope. For instance, the simple placement-mastery decision problem may be important in such areas as psychotherapy in which it can be expected that patients react differentially to a certain kind of therapy and the most promising therapy is followed by an end-of-therapy test.

The Placement-Mastery Decision Problem

In placement decisions several alternative treatments are available and it is the decision maker's task to assign individuals to the most promising treatment on the basis of their test scores. All subjects are administered the same test and the success of each treatment is measured by the same criterion. Figure 1 shows a flowchart of an ISS for the case of two instructional treatments in which the treatment assignment is followed by a mastery test. A test on the basis of which it is decided whether the student has mastered the instructional treatment sufficiently so that (s)he may proceed with the next treatment, or has to relearn the treatment and prepare him(her)self for a new test.

Insert Figure 1 about here

In the following, we shall suppose that in the placement-mastery decision problem the total population can be separated into g ($g \geq 2$) subpopulations reacting

differently to the test items. Let X be the placement test score variable, Y the mastery test score variable, and let the true score variable T underlying Y denote the criterion common to the treatments j (j = 0,1), respectively. The variables X, Y, and T will be considered to be continuous.

We consider a hypothetical experiment consisting of a population of students being exposed to each of both possible treatments but where the students are "brain-washed" so that the effects of one treatment do not interfere with those of another. (The actual experiment needed for parameter estimation and in which different samples of students are randomly assigned to the treatments will be described later on).

Furthermore, it is supposed that the relation between the measurements X, the measurements Y after treatment j, and the criterion T, can be represented for each population i (i = 1,2,....,g) by a joint probability function $\Omega_{ji}(x,y,t)$. Since the treatment is between the placement and the mastery test, it will influence the relation between X, Y and T, and this relation can be expected to assume a different shape for each treatment. This is indicated by the index j in $\Omega_{ji}(x,y,t)$. However, because the placement test is administered previous to the treatments, the marginal probability function of X in subpopulation i is the same for both treatments and will be denoted by $q_i(x)$. The above experiment being executed, the placement-mastery decision problem now consists of setting simultaneously cut-off scores x_{ci} and y_{ci} such that, given the value of t_c, the overall

expected utility is maximized. It should be stressed that, although the nature of the decisions shown in Figures 1 and 2 is sequential, the cut-off scores x_{ci} and y_{ci} are optimized simultaneously using data coming from the above experiment.

The presence of populations reacting differently to test items imply also different cut-off scores for each population (Gross & Su, 1975; Petersen & Novick, 1976). Therefore, let x_{ci} and y_{ci} denote the cut-off scores for subpopulation i on the observed test score variables X and Y, respectively. However, the cut-off score t_c on the criterion score T separating "true masters" from "true nonmasters" is assumed to be equal for each population. Note that, due to the presence of different populations reacting differently to test items, different probability functions for each population should be assumed (Gross & Su, 1975; Petersen & Novick, 1976).

In this paper, we consider only monotone decision rules δ: students with test scores above a certain cutting point are considered "suitable" and "not suitable" otherwise. For the decision network of Figure 1 they can be defined in the following way:

$$(1) \qquad \delta(X, Y) = \begin{cases} a_{00} & \text{for } X < x_{ci},\ Y < y_{ci} \\ a_{01} & \text{for } X < x_{ci},\ Y \geq y_{ci} \\ a_{10} & \text{for } X \geq x_{ci},\ Y < y_{ci} \\ a_{11} & \text{for } X \geq x_{ci},\ Y \geq y_{ci}, \end{cases}$$

where a_{jh} stands for the action either to retain (h = 0) or

advance (h = 1) a student who is assigned to treatment j (j = 0,1). The problem of setting optimal cutting scores x_{ci} and y_{ci}, given the value of t_c, now amounts to selecting a monotone decision rule which maximizes overall expected utility. However, the restriction to a subset of all possible rules in our paper is only correct if there are no nonmonotone rules with higher expected utility. The conditions under which the subclass of monotone rules is essentially complete, i.e., that for any nonmonotone rule there is a monotone rule that is at least as good (Ferguson, 1967, Sec.6.1; Karlin & Rubin, 1956) will be examined in the next section.

Monotonicity Conditions

For the elementary decisions the monotonicity conditions are known, and a monotone solution exists. The first condition is that the probability model relating observed test score variable Z to true score variable T has monotone likelihood ratio (MLR) in t, i.e. it is required that the ratio of likelihoods $f(z|t_2)/f(z|t_1)$ is nondecreasing in z for any $t_1 < t_2$. Second, the actions should be ordered such that for each two adjacent actions the utility functions possess at most one point of intersection.

To guarantee that a monotone solution of the combined decision problem exists, it is assumed that, in addition to the conditions of MLR and monotone utility, the following conditions hold in each subpopulation i for the probability

functions $v_{ji}(t|x)$, $p_{ji}(x,y|t)$, and $n_{ji}(y|x)$ of the distributions of T given X =x, (X,Y) given T = t, and Y given X = x under treatment j (j = 0,1), respectively:

(2) For any $x_1 < x_2$, the ratio of likelihoods $v_{0i}(t|x_2)/v_{1i}(t|x_1)$ is nondecreasing in t,

(3) For any $t_1 < t_2$, the ratio of likelihoods $p_{ji}(x,y|t_2)/p_{ji}(x,y|t_1)$ is nondecreasing in each of its arguments, that is, MLR in each of its arguments,

(4) For any $x_1 < x_2$, the ratio of likelihoods $n_{1i}(y|x_2)/n_{0i}(y|x_1)$ is nondecreasing in y.

After the utility function of the combined decision problem has been specified in the next section, and it has been indicated how the condition of monotone utility applies to this function, it will be proved that the above-mentioned conditions are sufficient for a monotone solution to exist.

An Additive Representation of the Combined Utility Function

Formally, a utility function $u_{jhi}(t)$ describes all costs and benefits involved when action a_{jh} (j,h = 0,1) is taken for the student from subpopulation i whose true score is t. The decision-maker may have different utilities associated with

different populations (Gross & Su, 1975; Petersen & Novick, 1976). Hence, in addition to separate probability distributions, the decision-maker has to specify explicitly his/her utility function for each subpopulation separately.

In the Introduction, it was remarked that one of the main advantages of the simultaneous approach was that more realistic utility structures could be used. This is nicely demonstrated by defining the utility structure of the combined decision problem as an additive function of the following form:

$$(5) \qquad u_{jhi}(t) = w_1 u_{jip}(t) + w_2 u_{him}(t),$$

where $u_{jip}(t)$, $u_{him}(t)$, w_1, and w_2 represent the utility functions for the separate placement and mastery decisions, and two nonnegative weights respectively. The utility functions $u_{jip}(t)$ and $u_{him}(t)$ are assessed separately and then brought onto the same scale by use of the weights w_1 and w_2. A set of conditions sufficient for the existence of an additive value function may be found in Fishburn (1982), French (1986), Keeney and Raiffa (1976, Chapt. 3), and Krantz et al. (1971). Since utility must be measured at most on an interval scale, the utility function of (5) can always be rescaled (normalized) as follows:

$$(6) \qquad u_{jhi}(t) = w u_{jip}(t) + (1-w) u_{him}(t),$$

where the weight w should satisfy $0 \leq w \leq 1$. The utility

function $u_{jhi}(t)$ now takes the following form:

$$(7) \qquad u_{hji}(t) = \begin{cases} wu_{0ip}(t) + (1-w)u_{0im}(t) & \text{for } j=0,\ h=0 \\ wu_{0ip}(t) + (1-w)u_{1im}(t) & \text{for } j=0,\ h=1 \\ wu_{1ip}(t) + (1-w)u_{0im}(t) & \text{for } j=1,\ h=0 \\ wu_{1ip}(t) + (1-w)u_{1im}(t) & \text{for } j=1,\ h=1. \end{cases}$$

It is reasonable to assume that the utility for granting mastery status is a nondecreasing function of a student's true score t, and the utility for denying mastery status is nonincreasing. Hence, the difference of the two utilities, $u_{1im}(t)-u_{0im}(t)$, is a nondecreasing function of t. In the following, we shall suppose that the treatments have been ordered in such a way that treatment 1 can be considered as the "higher" treatment. In general, students with high test scores on the placement test will be assigned to treatment 1, and vice versa. For instance, treatment 1 may contain less examples and exercises than treatment 0. Then, we may assume that $u_{1ip}(t)$ and $u_{0ip}(t)$ are nondecreasing and nonincreasing functions of t, respectively. Thus, it also follows that $(u_{1ip}(t) - u_{0ip}(t))$ is a nondecreasing function of t. Finally, since the difference of the two utilities for the separate decisions are nondecreasing functions of t, the condition of monotone utility is fulfilled for the utility functions of both separate decisions.

Maximization of Overall Expected Utility

As noted earlier, the optimal procedure from a Bayesian point of view is to look for a rule that maximizes the overall expected utility. Since we may confine ourselves to monotone rules, the expected utility of a random student from subpopulation i for the simultaneous approach is given by

$$
\begin{aligned}
(8)\qquad & E[u_{jhi}(T)\,|\,x_{ci}, y_{ci}] \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{x_{ci}}\int_{-\infty}^{y_{ci}} u_{00i}(t)\,\Omega_{0i}(x,y,t)\,dt\,dx\,dy + \\
&\quad \int_{-\infty}^{\infty}\int_{-\infty}^{x_{ci}}\int_{y_{ci}}^{\infty} u_{01i}(t)\,\Omega_{0i}(x,y,t)\,dt\,dx\,dy + \\
&\quad \int_{-\infty}^{\infty}\int_{x_{ci}}^{\infty}\int_{-\infty}^{y_{ci}} u_{10i}(t)\,\Omega_{1i}(x,y,t)\,dt\,dx\,dy + \\
&\quad \int_{-\infty}^{\infty}\int_{x_{ci}}^{\infty}\int_{y_{ci}}^{\infty} u_{11i}(t)\,\Omega_{1i}(x,y,t)\,dt\,dx\,dy.
\end{aligned}
$$

Substituting the additive utility function of (7) into (8), and rearranging terms, yields

$$
\begin{aligned}
(9)\qquad & E[u_{jhi}(T)\,|\,x_{ci}, y_{ci}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0[u_{00i}(T)\,|\,x,y]\,k_{0i}(x,y)\,dx\,dy + \\
&\quad \int_{x_{ci}}^{\infty} (E_1[u_{10i}(T)\,|\,x] - E_0[u_{00i}(T)\,|\,x])\,q_i(x)\,dx + \\
&\quad (1-w)\left(\int_{y_{ci}}^{\infty} (E_0[u_{11m}(t) - u_{01m}(t)\,|\,y])\,s_{0i}(y)\,dy + \right.
\end{aligned}
$$

$$\int_{x_{ci}}^{\infty}\int_{y_{ci}}^{\infty} q_i(x)\{\{E_1[u_{1im}(t)-u_{0im}(t)\,|x,y]\}n_{1i}(y|x) - \{E_0[u_{1im}(t)-u_{0im}(t)\,|x,y]\}n_{0i}(y|x)\}dxdy\},$$

where $s_{ji}(y)$ and $k_{ji}(x,y)$ denote the probability function of Y and (X,Y) in subpopulation i under treatment j, respectively, and where E_j indicates that the expectation has been taken over a distribution indexed by j (j = 0,1).

Now, the decision procedure is viewed as a series of separate decisions, each of which involves one random student from the total population. Furthermore, it is assumed that the overall expected utility for the simultaneous approach, $EU_{sim}(x_{c1},y_{c1},\ldots,x_{cg},y_{cg})$, is found by summing the expected utility for the simultaneous approach of a random student over all students. Under these assumptions, it follows that the overall expected utility for the simultaneous approach can be written as:

$$(10)\qquad EU_{sim}(x_{c1},y_{c1},\ldots,x_{cg},y_{cg}) = \sum_{i=1}^{g} p_i E[u_{jhi}(T)\,|x_{ci},y_{ci}],$$

where p_i, $\sum_{i=1}^{g} p_i = 1$, is the proportion of students from subpopulation i in the total population.

Sufficiency of the Monotonicity Conditions

There is a theorem in decision theory (see e.g., Chuang, Chen, and Novick, 1981) stating that $E[u(\theta)\,|z)$ is a

nondecreasing function of z if $f(z|\theta)$ has MLR and $u(\theta)$ is a nondecreasing function of θ. We will refer to this property as monotone expected utility (MEU).

The utility associated with action a_{10}, $u_{10i}(t)$, may either be a nondecreasing or a nonincreasing function of t. Since $(-u_{00i}(t))$ is a nondecreasing function of t, and using the assumption of MLR of the probability function X given T=t, it follows by applying the MEU theorem that in the case of $u_{10i}(t)$ being a nondecreasing function of t, i.e., $-\frac{d}{dt}u_{0im}(t)/(\frac{d}{dt}[u_{1ip}(t)-u_{0im}(t)]) \leq w \leq 1$, $(E_1[u_{10i}(T)|x] - E_0[u_{00i}(T)|x])$ is a nondecreasing function of x. Furthermore, since $[u_{10i}(t) - u_{00i}(t)] = w[u_{1ip}(t)-u_{0ip}(t)]$ and $(-u_{00i}(t))$ are nondecreasing functions of t, and using the assumption of MLR of the probability function of X given T=t, it follows by applying the MEU theorem that both $(E_0[u_{10i}(T)|x] - E_0[u_{00i}(T)|x])$ and $(-E_0[u_{00i}(T)|x])$ are nondecreasing functions of x. In the case of $u_{10i}(t)$ being a strictly decreasing function of t, i.e., $0 \leq w < -\frac{d}{dt}u_{0im}(t)/(\frac{d}{dt}u_{1ip}(t) - u_{0im}(t)]$, and multiplying $E_0[u_{10i}(T)|x]$ by $v_{1i}(t|x)/v_{0i}(t|x)$, it now follows under condition 2, which implies that $v_{1i}(t|x)/v_{0i}(t|x)$ is nonincreasing in t, that $(E_1[u_{10i}(T)|x] - E_0[u_{00i}(T)|x])$ is a nondecreasing function of x. Thus, for $u_{10i}(t)$ being a nondecreasing as well as being a strictly increasing function of t, $(E_1[u_{10i}(T)|x] - E_0[u_{00i}(T)|x])$ is a nondecreasing function of x. Using $q_i(x) \geq 0$, it then follows that the integrand of the second term of (9) is a nondecreasing function of x.

With the assumed MLR of the probability function of Y given T = t, and using $[u_{1im}(t)-u_{0im}(t)]$ is nondecreasing in t and $s_{0i}(y) \geq 0$, it follows by the MEU theorem that the integrand of the third term of (9) is a nondecreasing function of y.

Furthermore, since $[u_{1im}(t)-u_{0im}(t)]$ is a nondecreasing function in t, it follows under condition 3 by applying the MEU theorem that $(E_1[u_{1im}(t)-u_{0im}(t)\,|x,y])$ and $-(E_0[u_{1im}(t)-u_{0im}(t)\,|x,y])$ are nondecreasing and nonincreasing in each of their arguments x and y, respectively. Multiplying $-(E_0[u_{1im}(t) - u_{0im}(t)\,|x,y])$ by $n_{0i}(y|x)/n_{1i}(y|x)$, it follows under condition 4, which implies that $n_{0i}(y|x)/n_{1i}(y|x)$ is nondecreasing in y, that $-(E_0[u_{1im}(t) - u_{0im}(t)\,|x,y])\ n_{0i}(y|x)/n_{1i}(y|x)$ is nondecreasing in each of its arguments. Multiplying both $(E_1[u_{1im}(t) - u_{0im}(t)\,|x,y])$ and $-(E_0[u_{1im}(t)-u_{0im}(t)\,|x,y])$ $n_{0i}(y|x)/n_{1i}(y|x)$ by $n_{1i}(y|x)$, and using $n_{1i}(y|x)$, $q_i(x) \geq 0$, it now applies that the integrand of the fourth term of (9) is a nondecreasing function in each of its arguments.

It finally follows that $E[u_{jhi}(T)\,|x_{ci},y_{ci}]$, and hence Expression 10, possesses an absolute maximum because the first term is a constant, $(1-w) \geq 0$, and the integrand of each term is nondecreasing in x, y, or in each of its arguments x and y. In each new application it must be checked if the conditions of MLR, monotone utility, and (2)-(4) hold. Checking for the monotonicity conditions will be considered below after the (conditional) probability functions appearing in (9) have been specified.

Optimizing Cutting Scores for Quota-Free Placement

With quota-free placement, there is no constraint on the number of students that can be assigned to one of the treatments. Therefore, the values of the optimal cutting scores, say x'_{ci} and y'_{ci}, which maximize Expression 10, can be obtained by maximizing Expression 9 for each subpopulation i separately. The optimal rule can be derived by differentiating $E[u_{jhi}(T)|x_{ci},y_{ci}]$ with respect to x_{ci} and y_{ci}, setting the resulting expressions equal to zero, and solving for x_{ci} and y_{ci}.

First, differentiating $E[u_{jhi}(T)|x_{ci},y_{ci}]$ with respect to y_{ci}, and assuming $w \neq 1$, results in

$$(11) \quad s_{0i}(y_{ci})\{E_0[u_{1im}(t)-u_{0im}(t)|y_{ci}]\} +$$

$$s_{1i}(y_{ci})\int_{x_{ci}}^{\infty}\{E_1[u_{1im}(t)-u_{0im}(t)|x,y_{ci}]\}z_{1i}(x|y_{ci})dx -$$

$$s_{0i}(y_{ci})\int_{x_{ci}}^{\infty}\{E_0[u_{1im}(t)-u_{0im}(t)|x,y_{ci}]\}z_{0i}(x|y_{ci})dx = 0,$$

where $z_{ji}(x|y_{ci})$ denotes the probability function of X given $Y = y_{ci}$ under treatment j in subpopulation i ($j = 0,1$). Similarly, differentiating $E[u_{jhi}(T)|x_{ci},y_{ci}]$ with respect to x_{ci}, and using $q_i(x) > 0$ (the possibility of $q_i(x) = 0$ will be ignored), yields

$$(12) \qquad E_1[u_{10i}(T)|x_{ci}] - E_0[u_{00i}(T)|x_{ci}] +$$

$$(1-w)\int_{y_{ci}}^{\infty} (\{E_1[u_{1im}(t)-u_{0im}(t)|x_{ci},y]\}n_{1i}(y|x_{ci}) -$$

$$\{E_0[u_{1im}(t)-u_{0im}(t)|x_{ci},y]\}n_{0i}(y|x_{ci}))dy = 0.$$

Since the system of Equations 11 and 12 cannot be solved analytically for x_{ci} and y_{ci}, the determination of the optimal cutting scores may be carried out via numerical approximation procedures such as the Newton iterative algorithm for solving nonlinear equations. However, before proceeding with this procedure, it is necessary to specify the probability functions, regression functions, and utility functions appearing in (11) and (12).

Bivariate Normal Model

In the following, we shall suppose that the variables X and Y have possibly different bivariate normal distributions under both treatments in each subpopulation i. Let ρ_{ji} denote the population correlation between X and Y under treatment j in subpopulation i, and let x_N and y_{Nj} denote the standardized scores of X and Y under treatment j (j = 0,1), respectively. Then it can be shown (see e.g. Johnson & Kotz, 1970) that for the standardized bivariate normal distribution the conditional distribution of X_N given $Y_{Nj} = y_{Nj}$ is normal with expected value $\rho_{ji}y_{Nj}$ and variance $(1-\rho_{ji}^2)$. Likewise, $n_{ji}(y_N|x_N)$ is normal with expected value $\rho_{ji}x_N$ and variance $(1-\rho_{ji}^2)$.

The regression functions $E_{ji}(T|x)$ and $E_{ji}(T|x,y)$ of T on

x and T on x and y under treatment j, are assumed to be linear in each subpopulation i; that is, they can be written as $\theta_{ji}+\Gamma_{ji}x$ and $\alpha_{ji}+\beta_{ji}x+\tau_{ji}y$, respectively. Using results from classical test theory, it follows that the regression coefficients can be written as :

(13)
$$\Gamma_{ji} = \rho_{ji}(\sigma_{Yji}/\sigma_{Xi})$$
$$\theta_{ji} = \mu_{Yji} - \Gamma_{ji}\mu_{Xi}$$
$$\beta_{ji} = (\sigma_{Yji}/\sigma_{Xi})\,((\rho_{ji}-\rho_{Yji}\rho_{ji})/(1-\rho_{ji}^2))$$
$$\tau_{ji} = (\rho_{Yji}-\rho_{ji}^2)/(1-\rho_{ji}^2)$$
$$\alpha_{ji} = -\mu_{Xi}\beta_{ji} + \mu_{Yji}(1-\tau_{ji}),$$

μ_{Yji}, μ_{Xi}, σ_{Yji}, σ_{Xi}, and ρ_{Yji} being the population means of Y and X, the population standard deviations of Y and X, and the reliability coefficient of Y under treatment j (j = 0,1) in subpopulation i, respectively. Assuming also linear regression for T on y under both treatments in each subpopulation i, and using Kelley's regression line (Lord & Novick, 1968, p.55), it follows

(14)
$$E_{ji}(T|y) = \rho_{Yji}\,y + (1-\rho_{Yji})\mu_{Yji}.$$

Furthermore, assuming homoscedascity, it also follows from classical test theory that:

$$(15) \qquad \begin{aligned} \mathrm{var}_{ji}(T|x) &= (\rho_{Yji}-\rho_{ji}^2)\sigma_{Yji}^2 \\ \mathrm{var}_{ji}(T|y) &= \sigma_{Yji}^2\rho_{Yji}(1-\rho_{Yji}) \\ \mathrm{var}_{ji}(T|x,y) &= \sigma_{Yji}^2(\rho_{Yji}-\rho_{ji}^2-\rho_{Yji}^2+\rho_{ji}^2\rho_{Yji})/ \\ &\qquad (1-\rho_{ji}^2). \end{aligned}$$

Having defined the probability and regression functions, we now are in a position for checking if the monotonicity conditions are satisfied. First, it can be noticed that, since the probability functions of T given X=x, and T given X=x and Y=y in subpopulation i under treatment j are normal, they belong to the exponential family, and hence, they do have MLR and MLR in each of its arguments, respectively (see e.g., Chuang, Chen & Novick, 1981). Thus, monotonicity condition 3 is fulfilled.

Furthermore, it can be shown (see e.g., Lehmann, 1959, sect. 3.13) that a necessary and sufficient condition for the likelihood ratios in (2) and (4) to be nondecreasing in t and in y, respectively, is that the mixed second derivative of the natural logarithm of these ratios exists and is nonnegative. Differentiating these ratios of normal distributions, it then applies that the following set of conditions should hold for conditions 2 and 4 respectively.

$$(16) \qquad \begin{aligned} &\Gamma_{0i}/\mathrm{var}_{0i}(T|x) \geq \Gamma_{1i}/\mathrm{var}_{1i}(T|x) \\ &\rho_{1i}\sigma_{Y0i}/(1-\rho_{1i}^2) \geq \rho_{0i}\sigma_{Y1i}/(1-\rho_{0i}^2). \end{aligned}$$

Linear Utility

Although utility functions can be empirically assessed without making any assumptions about the form of the utility functions (e.g. Vrijhof, Mellenbergh & van den Brink, 1983), usually the form of the utility function is specified on a priori grounds. In statistical decision theory several forms of the utility functions have been adopted. In the present paper, it will be assumed that the utility structures for both the separate decisions are linear functions of the criterion variable t. This utility structure seems to be a realistic representation of the utilities actually incurred in many decision making situations. In a recent study, for instance, it was shown by van der Gaag (1989) that many empirical utility structures could be approximated by linear functions. For other frequently used utility functions and their (dis)advantages, refer to Lindley (1976), Novick and Lindley (1978), Swaminathan, Hambleton and Algina (1975), and van der Linden (1981).

Mellenbergh and van der Linden (1981) and van der Linden and Mellenbergh (1977) proposed a linear utility function for the separate decisions. They can in the case of multiple populations be defined in the following way:

$$(17) \qquad u_{jip}(t) = \begin{cases} b_{0ip}(t_c - t) + d_{0ip} & \text{for } X < x_{ci} \\ b_{1ip}(t - t_c) + d_{1ip} & \text{for } X \geq x_{ci} \end{cases}$$

$$(18) \qquad u_{him}(t) = \begin{cases} b_{0im}(t_c - t) + d_{0im} & \text{for } Y < y_{ci} \\ b_{1im}(t - t_c) + d_{1im} & \text{for } Y \geq y_{ci}, \end{cases}$$

where b_{0ip}, b_{1ip}, b_{0im}, and $b_{1im} > 0$.

For each action, this function consists of a constant term and a term proportional to the difference between the criterion performance t of a student and the minimum level of satisfactory criterion performance t_c. The parameters d_{jip} and d_{him} (j,h = 0,1) can, for example, represent the constant amount of costs of following treatment j and the costs of testing, respectively, and will in that case have a nonpositive value. The condition b_{0ip}, $b_{1ip} > 0$ is equivalent to the assumption that for assigning students to treatment 0 and 1, utility is a strictly decreasing and increasing function of t, respectively. Likewise, b_{0im}, $b_{1im} > 0$ expresses the assumption that the utility for failing and advancing the mastery test is a strictly decreasing and increasing function of t, respectively.

As Gross & Su (1975) and Petersen & Novick (1976) pointed out, the question whether decision rules are fair to the various subpopulations which can be distinguished depend within a decision-theoretic framework only on the chosen utilities. From this point of view, separate parameter values might be chosen in the linear utility model to allow for the fact that the students might belong to a disadvantaged or advantaged population (see also Mellenbergh & van der Linden, 1981). Suppose, for example, that population 2 is considered more advantaged than 1. Furthermore, it is assumed that incorrect decisions are considered worse for population 1 than for 2, while correct decisions are considered more valuable for population 1 than for 2. If so, b_{j1p} and b_{h1m}

could be set higher than b_{j2p} and b_{h2m}, respectively, for every value of t (j,h = 0,1).

Substituting the assumed linear regression functions, bivariate normal probability functions, and linear utility functions in (11), it follows by integration over x:

$$(19)\qquad f(x_{ci},y_{ci}) = ((b_{1im}+b_{0im})(\rho_{Y0i}y_{ci}+(1-\rho_{Y0i})\mu_{Y0i}-t_c)+d_{1im}-d_{0im})\,\varphi[y_{N0ci}]/\sigma_{Y0i} + \sum_{j=0}^{1} a_j f_j(x_{ci},y_{ci}) = 0,$$

where $a_j = 2j - 1$,

$$f_j(x_{ci},y_{ci}) = ((b_{1im}+b_{0im})(\alpha_{ji}+\beta_{ji}\mu_{Xi}+\tau_{ji}y_{ci}+\beta_{ji}\sigma_{Xi}\rho_{ji}y_{Njci}-t_c)+d_{1im}-d_{0im})(1-\Phi[z_{xyjNci}]) + (b_{1im}+b_{0im})\beta_{ji}\sigma_{Xi}\sqrt{(1-\rho_{1i}^2)}\,\varphi[z_{xyjNci}]\,\varphi[y_{Njci}]/\sigma_{yji},$$

and $\Phi[.]$, $\varphi[.]$, and z_{xyjNci} denote the standard normal distribution function, standard normal density, and $(x_{Nci}-\rho_{ji}y_{Njci})/\sqrt{(1-\rho_{ji}^2)}$, respectively. Similarly, it follows that (12) can be replaced by

$$(20)\qquad g(x_{ci},y_{ci}) = \sum_{j=0}^{1} (w(b_{jip}(\theta_{ji}+\Gamma_{ji}x_{ci}-t_c)+a_jd_{jip}) + (1-w)(a_j(b_{0im}(t_c-\theta_{ji}-\Gamma_{ji}x_{ci})+d_{0im}) + f'_j(x_{ci},y_{ci}))) = 0,$$

where

$$f'_j(x_{ci}, y_{ci}) = [(b_{1im}+b_{0im})(\alpha_{ji}+\tau_{ji}\mu_{Yji}+\beta_{ji}x_{ci} + \tau_{ji}\sigma_{Yji}\rho_{ji}x_{Nci}-t_c) + d_{1im}-d_{0im}](1-\Phi[z_{yjxNci}]) + (b_{1im}+b_{0im})\tau_{ji}\sigma_{Yji}\sqrt{(1-\rho^2_{ji})}\,\varphi[z_{yjxNci}],$$

with $z_{yjxNci} = (y_{Njci}-\rho_{ji}x_{Nci})/\sqrt{(1-\rho^2_{ji})}$.

To apply Newton's iterative procedure, the partial derivatives of the left-hand side of (19) and (20) are needed. They are given as

$$(21)\qquad \frac{\partial}{\partial x_{ci}} f(x_{ci}, y_{ci}) =$$

$$-\sum_{j=0}^{1} (a_j[\sqrt{(1-\rho_{1i}^2)}\sigma_{Y1i}]^{-1}\,\varphi[y_{N1ci}][(b_{1im}+b_{0im})(\alpha_{ji}+\beta_{ji}x_{ci}+\tau_{ji}y_{ci}-t_c)+d_{1im}-d_{0im}]\varphi[z_{xjNci}]/\sigma_{Xci},$$

$$(22)\qquad \frac{\partial}{\partial y_{ci}} f(x_{ci}, y_{ci}) =$$

$$-[(b_{1im}+b_{0im})(\rho_{Y0i}y_{ci}+(1-\rho_{Y0i})\mu_{Y0i}-t_c)+d_{1im}-d_{0im})\, y_{N0ci}\varphi[y_{N0ci}]/\sigma_{Y0i}^2+(b_{1im}+b_{0im})\rho_{Y0i}\varphi[y_{N0ci}]/\sigma_{Y0i}$$

$$-\sum_{j=0}^{1} (f_j(x_{ci}, y_{ci})\, y_{Njci}\varphi[y_{Njci}]/\sigma_{Yji}^2 +$$

$$(a_j(b_{1im}+b_{0im})\,[(\tau_{ji}\sigma_{Yji}+\beta_{ji}\rho_{ji}\sigma_{Xi})(1-\Phi[z_{xjNci}])+\rho_{ji}[\sqrt{(1-\rho_{ji}^2)}]^{-1}\varphi[z_{xjNci}](\alpha_{ji}+\beta_{ji}x_{ci}+\tau_{ji}y_{ci}-t_c)] + (d_{1im}-d_{0im})\rho_{ji}[\sqrt{(1-\rho_{ji}^2)}]^{-1}\,\varphi[z_{xjNci}])\varphi[y_{Njci}]/\sigma_{Yji}^2),$$

$$(23)\qquad \frac{\partial}{\partial x_{ci}}\, g(x_{ci}, y_{ci}) =$$

$$w(b_{1ip}\Gamma_{1i}+b_{0ip}\Gamma_{0i})+(1-w)b_{0im}(\Gamma_{0i}-\Gamma_{1i}) +$$

$$\sum_{j=0}^{1} a_j(1-w)\{(b_{1im}+b_{0im})/\sigma_{Xi}\{(\beta_{ji}\sigma_{Xi}+\tau_{ji}\sigma_{Yji}\rho_{ji})$$

$$(1-\Phi[z_{yjxNci}])+\rho_{ji}[\sqrt{(1-\rho_{ji}^2)}]^{-1}\ \varphi[z_{yjxNci}]$$

$$(\alpha_{ji}+\beta_{ji}x_{ci}+\tau_{ji}y_{ci}-t_c)\}+(d_{1im}-d_{0im})\rho_{ji}[\sqrt{(1-}$$

$$\rho_{ji}^2)\sigma_{Xi}]^{-1}$$

$$\varphi[z_{yjxNci}],$$

$$(24)\qquad \frac{\partial}{\partial x_{ci}}\, g(x_{ci}, y_{ci}) =$$

$$\sum_{j=0}^{1} a_j(1-w)\{[\sqrt{(1-\rho_{ji}^2)}\sigma_{Yji}]^{-1}\ (-(b_{1im}+b_{0im})$$

$$(\alpha_{ji}+\beta_{ji}x_{ci}+\tau_{ji}y_{ci}-t_c) + (d_{0im}-d_{1im})\}\varphi[z_{yjxNci}].$$

Inserting Equations 19 until 24 into Newton's procedure, and using the property that the standard normal distributions appearing in this system of Equations can be approximated by logistic functions with a scale parameter equal to 1.7 (Lord & Novick, 1968, sect. 17.2), one obtains the optimal cutting scores x'_{ci} and y'_{ci}. The algorithm is implemented in a computer program called NEWTON.

Derivation of Optimal Separate Decisions

The expected utility of a random student for the separate

mastery decision from subpopulation i, $E[u_{him}(T)|y_{ci}]$, follows immediately from (9) by realizing that both treatments can be thought to coincide in this case implying that $n_{0i}(y|x) = n_{1i}(y|x)$ and $E_0[.] = E_1[.]$. Also, the combined additive utility function, $u_{jhi}(t)$, can be replaced in this case by $[(1-w)u_{him}(t)]$ implying that amongst others $u_{10i}(t) = u_{00i}(t) = (1-w)u_{0im}$. Furthermore, the placement test score variable X can be thought to coincide with the true score variable T in this case implying that the first term of (9) reduces to $(1-w)\int_{-\infty}^{\infty} E_0[u_{0im}(T)|y]s_{0i}(y)dy$. Substituting the above-mentioned equalities into (9), results in

$$(25) \qquad E[u_{him}(T)|y_{ci}] = (1-w)\{\int_{-\infty}^{\infty} E_0[u_{0im}(T)|y]s_{0i}(y)dy + \int_{y_{ci}}^{\infty}(E_0[u_{1im}(t) - u_{0im}(t)|y]]s_{0i}(y)dy).$$

The optimal separate mastery scores, y'_{ci}, can be derived again by differentiating $E[u_{him}(T)|y_{ci}]$ with respect to y_{ci}, setting the resulting expression equal to zero, and solving for y_{ci}. Doing so, and using $s_{0i}(y_{ci})>0$ and $w \neq 1$, results in

$$(26) \qquad E_0[u_{1im}(t)-u_{0im}(t)|y_{ci}] = 0,$$

which yields the same optimal cutting score y'_{ci} as the one given by van der Linden and Mellenbergh (1977). For the

linear utility model (Expression 18), it follows from (26) that

$$(27) \qquad y'_{ci} = \mu_{Y0i} + (t_c - \mu_{Y0i} + (d_{0im} - d_{1im})/(b_{1im} + b_{0im}))/\rho_{Y0i}.$$

As an aside, it may be noted that Equation 26, and hence Equation 27, can also be derived immediately by substituting the equalities $s_{0i}(y_{ci}) = s_{1i}(y_{ci})$, $z_{0i}(x|y_{ci}) = z_{1i}(x|y_{ci})$, and $E_0[.] = E_1[.]$ into (11).

Next, the expected utility of a random student for the separate placement decision from subpopulation i, $E[u_{jip}(T)|x_{ci}]$, can easily be derived from (9) by realizing that in the combined additive utility function, $u_{jhi}(t)$, the term $[(1-w)u_{him}(t)]$ vanishes in this case implying that the third term of (9) also vanishes, and the second term of (9) reduces to $[w \int_{x_{ci}}^{\infty} (E_1[u_{1ip}(T)|x] - E_0[u_{0ip}(T)|x])q_i(x)dx]$. Furthermore, the mastery test score variable Y can be thought to coincide with the true score variable T in this case implying that the first term of (9) reduces to $w \int_{-\infty}^{\infty} E_0[u_{0ip}(T)|x]q_i(x)dx$. Substituting the above-mentioned results into (9), yields

$$(28) \qquad E[u_{jip}(T)|x_{ci}] = w(\int_{-\infty}^{\infty} E_0[u_{0ip}(T)|x]q_i(x)dx + \int_{x_{ci}}^{\infty} (E_1[u_{1ip}(T)|x] - E_0[u_{0ip}(T)|x])q_i(x)dx).$$

The expected utility for a random student from

subpopulation i for the separate approach is found by summing $E[u_{him}(T)|y_{ci}]$ and $E[u_{jip}(T)|x_{ci}]$. Finally, analogously to the simultaneous approach, the overall expected utility for the separate approach, $EU_{sep}(x_{c1}, y_{c1}, \ldots, x_{cg}, y_{cg})$, is found by summing the expected utility for the separate approach of a random student over all students yielding

$$(29) \qquad EU_{sep}(x_{c1},y_{c1},\ldots,x_{cg},y_{cg}) = \sum_{i=1}^{g} p_i\{E[u_{him}(T)|y_{ci}]+E[u_{jip}(T)|x_{ci}]\}.$$

The optimal separate placement cutting scores, y'_{ci}, follow now again by differentiating $E[u_{jip}(T)|x_{ci}]$ with respect to x_{ci}, setting the resulting expression equal to zero, and solving for x_{ci}. Doing so, and using $q_i(x) > 0$ and $w \neq 0$, results in

$$(30) \qquad E_1[u_{1ip}(T)|x_{ci}]-E_0[u_{0ip}(TI|x_{ci}] = 0.$$

Also, this optimal solution agrees with the one reached by van der Linden (1981). Adopting the linear utility model from Expression 17 in (30), it applies that

$$(31) \qquad x'_{ci} = (d_{0ip}-d_{1ip}+t_c(b_{1ip}+b_{0ip})-b_{1ip}\theta_{1i}-b_{0ip}\theta_{0i})/(b_{1ip}\Gamma_{1i}+b_{0ip}\Gamma_{0i}).$$

Equation 30, and hence Equation 31, can also easily be derived by putting $(1-w)u_{1im} = (1-w)u_{0im} = 0$ in (12).

As a final remark, it should be noted that the optimal separate cutting scores, unlike the expected utilities for the separate decisions, do not depend upon the value of w. On the other hand, the optimal cutting scores as well as the expected utilities for the simultaneous approach depend upon the value of w.

Solution for Quota-Restricted Placement

In the restricted placement situation only a fixed number of students can be assigned to each treatment. Confining ourselves to assignment decisions with two treatments, this constraint can be expressed as

$$(32) \qquad p_0 = \sum_{i=1}^{q} p_i [\text{Prob}(X \geq x_{ci})] = \sum_{i=1}^{q} p_i \left[\int_{x_{ci}}^{\infty} q_i(x)\,dx\right],$$

where $0 < p_0 < \sum_{i=1}^{q} p_i = 1$ represents the fixed proportion of all students that can be assigned to treatment 1.

Analogous to the quota-free model, optimal cutting scores x'_{ci} and y'_{ci} can be derived by maximization of the overall expected utility for the simultaneous approach from Equation 10 subject to (32). To solve this constrained optimization problem, first introduce the placement restriction into the function to be optimized (Equation 10) through a Lagrange multiplier λ:

$$(33) \qquad L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda) = \sum_{i=1}^{g} p_i E[u_i(T)\,|\,x_{ci},y_{ci}] + \lambda\Big(\sum_{i=1}^{g} p_i\Big[\int_{x_{ci}}^{\infty} q_i(x)\,dx\Big] - p_0\Big),$$

where λ is a constant. First, differentiating $L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda)$ with respect to y_{ci} and setting the resulting expression equal to zero yields the same solution as the solution for quota-free placement given by Equation 11. Next, differentiating of $L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda)$ with respect to λ, setting the resulting expression equal to zero yields, of course, Equation 32. Finally, differentiating $L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda)$ with respect to x_{ci}, setting the resulting expression equal to zero, and using p_i, $q_i(x) > 0$, yields

$$(34) \qquad E_1[u_{10i}(T)\,|\,x_{ci}] - E_0[u_{00i}(T)\,|\,x_{ci}] + \lambda + (1-w)\int_{y_{ci}}^{\infty} ((E_1[u_{11m}(t)-u_{01m}(t)\,|\,x_{ci},y]) n_{1i}(y|x_{ci}) - (E_0[u_{11m}(t)-u_{01m}(t)\,|\,x_{ci},y]) n_{0i}(y|x_{ci}))dy = 0.$$

Now, optimizing cutting scores for quota-restricted placement proceeds by substituting the assumed probability, regression, and utility functions into Equations 11, 32, and 34 and solving this system of Equations for the (2g + 1) unknown parameters x_{ci}, y_{ci}, and λ using Newton's iterative method again. For the linear utility model this results in the system of nonlinear Equations $f(x_{ci},y_{ci})$, (32), and

$g(x_{ci},y_{ci}) + \lambda$, respectively. Note that with quota-restricted placement, unlike quota-free placement, the optimal cutting scores x'_{ci} and y'_{ci} ($i = 1,\ldots,g$) are dependent upon each other. Also, the partial derivatives of the given system of nonlinear Equations is required again. The derivatives of $f(x_{ci},y_{ci})$ and $g(x_{ci},y_{ci})$ with respect to x_{ci} and y_{ci} were given already in the preceding sections. Furthermore, it can easily be derived that $\frac{\partial}{\partial x_{ci}}[g(x_{ci},y_{ci})+\lambda] = \frac{\partial}{\partial x_{ci}} g(x_{ci},y_{ci})$, $\frac{\partial}{\partial y_{ci}}[g(x_{ci},y_{ci})+\lambda] = \frac{\partial}{\partial y_{ci}} g(x_{ci},y_{ci})$, and $\frac{\partial}{\partial \lambda}[g(x_{ci},y_{ci})+\lambda] = 1$. Since $q_i(x)$ is a normal distribution with mean μ_{Xi} and variance σ_{Xi}^2 (see, e.g., Johnson & Kotz, 1972), it finally follows from (32) that

$$(35) \qquad \frac{\partial}{\partial x_{ci}}\Big(\sum_{i=1}^{g} p_i\Big[\int_{x_{ci}}^{\infty} q_i(x)dx\Big]-p_0\Big) = (-p_i/\sigma_{Xi})\varphi[x_{Nci}].$$

When there are allocation quota considerations, the set of given monotonicity conditions do not hold without modifications. Since λ is a constant, it follows directly from (11) and (34) that this set is also sufficient to guarantee that the integrand of each term in $L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda)$ is nondecreasing in x, y, or in each of its arguments x and y. In addition to these conditions, it should hold that $L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda)$ is a nondecreasing in λ or $\frac{\partial}{\partial \lambda} L(x_{c1},y_{c1},\ldots,x_{cg},y_{cg},\lambda) \geq 0$. Using (33), then the following extra monotonicity condition can be formulated

$$(36) \qquad \sum_{i=1}^{g} p_i \left[\int_{x_{ci}}^{\infty} q_i(x)\,dx \right] \geq p_0.$$

Since the above stated condition should hold for every value of x_{ci}, it follows that

$$(37) \qquad \sum_{i=1}^{g} p_i \left[\int_{x'_{ci,free}}^{\infty} q_i(x)\,dx \right] \geq p_0,$$

where $x'_{ci,free}$ denote the optimal placement cutting scores in the quota-free model. Furthermore, let $x'_{ci,quota}$ denote the optimal placement cutting scores in the restricted model, then it will generally hold that $x'_{ci,quota} > x'_{ci,free}$, because fewer students can be assigned to treatment 1 in the restricted model. Since the left-hand side of (36)) is a nondecreasing function in x_{ci} and is equal to p_0 when $x_{ci} = x'_{ci,quota}$, it follows that

$$\sum_{i=1}^{g} p_i \left[\int_{x_{ci}}^{\infty} q_i(x)\,dx \right] \geq p_0, \text{ for } x'_{ci,free} \leq x_{ci} \leq x'_{ci,quota}.$$

Hence, using $(x'_{ci,free}, y'_{ci,free})$ as a first approximation in Newton's iterative procedure guarantees that $L(x_{c1}, y_{c1}, \ldots, x_{cg}, y_{cg}, \lambda)$ reaches its maximum for one pair of cutting scores.

It should be noted that condition 37 can easily be seen to hold by realizing that if the left-hand side of condition 37 is less than p_0, this implies that fewer students are

assigned to treatment 1 than the number of places available. In applying the quota-restricted model, it must always be checked if condition 37 holds. If this is not the case, the optimal placement cutting scores of the unrestricted model should be used. To compute the optimal cutting scores with quota-restricted placement for the linear utility model, the computer program QUO-SIM has been developed.

Analogously to the quota-free model, the optimal separate cutting scores can easily be derived from Equations 11, 32, and 34 by imposing certain restrictions. The optimal separate mastery scores, y'_{ci}, follow again immediately from (11) by substituting the equalities $s_{0i}(y_{ci}) = s_1(y_{ci})$, $z_{0i}(x|y_{ci}) = z_{1i}(x|y_{ci})$, and $E_0[.] = E_1[.]$ into (11) yielding Equation 27 in case of the linear utility model.

However, unlike the optimal separate mastery decision, the optimal separate placement decision for the quota-restricted model differs from the quota-free model. The optimal separate placement scores, x'_{ci}, follow by first putting $(1-w)u_{0im}(t) = (1-w)u_{1im}(t) = 0$ in (34) yielding

$$(38) \qquad E_1[u_{1ip}(T)|x_{ci}] - E_0[u_{0ip}(T)|x_{ci}] + \lambda = 0.$$

Next, substituting the linear utility and linear regression functions into (38), results in

$$(39) \qquad x_{ci}(b_{1ip}\Gamma_{1i} + b_{0ip}\Gamma_{0i}) - (d_{0ip} - d_{1ip}) - t_c(b_{1ip} + b_{0ip}) + b_{1ip}\theta_{1i} + b_{0ip}\theta_{0i} + \lambda = 0.$$

Now, the optimal separate placement scores for the quota-restricted model are found by solving the nonlinear system of Equations 32 and 39 for the (g+1) unknown parameters x_{ci} and λ using Newton's iterative algorithm again. Analogously to the simultaneous approach, it should be noted that with quota-restricted placement, unlike quota-free placement, the optimal cutting scores $x'_{ci}(i=1,\ldots,g)$ are dependent upon each other. In order to apply Newton's iterative method to solve the given system of nonlinear Equations, the partial derivatives are required again. The derivatives of (32) with respect to x_{ci} were given already in (35). Finally, from (39) it can easily be verified that the derivatives of the left-hand side of (39) with respect to λ and x_{ci} are given by 1 and $(b_{1ip}\Gamma_{1i} + b_{0ip}\Gamma_{0i})$, respectively.

Finally, it should be noted that, analogously to the quota-free model, condition 37 should hold in order to guarantee that no fewer students are assigned to treatment 1 than the number of places available; that is, the Lagrangian function is nondecreasing in λ. In condition 37, $x'_{ci,free}$ denote now, of course, the optimal separate placement cutting scores of the quota-free model. If the resulting sum of products of the left-hand side of (37) is less than p_0, then the decision-maker should use again the optimal separate placement cutting scores of the unrestricted model.

A computer program called QUO-SEP has been written to obtain the optimal separate placement cutting scores for the quota-restricted model. To illustrate the models presented in this paper, a numerical example is given in the next section.

An Application of the Combined Decision Problem

The procedure for computing the optimal cutting scores was applied to a sample of 59 freshmen in medicine. Both the placement and mastery test was composed of 21 free-response items on elementary medical knowledge with test scores ranging from 0-100. The treatments 0 and 1 consisted of an interactive video (IV) and a computer-aided instructional (CAI) program, respectively. Since the IV-program contained more examples and exercises, treatment 1 was considered as the "higher" treatment.

Due to previous schooling, the total sample of 59 students could be separated with respect to elementary medical knowledge into a disadvantaged and an advantaged population of 31 and 28 students referred to as populations 1 and 2, respectively. The normal models assumed for the distributions X_i and Y_{ji} ($j = 0,1$; $i = 1,2$) showed a satisfactory fit to the test data for a Kolmogorov-Smirnov goodness-of-fit test.

The teachers of the course considered a student as having mastered the subject matter if (s)he could answer correctly at least 55% of the total domain of items. Therefore, t_c was fixed at 0.55.

The means, standard deviations, and correlations between X and Y were computed for each subpopulation under both treatments using the unbiased maximum likelihood estimates of the sample means, sample standard deviations, and sample

correlations, respectively. Furthermore, since the items were not scored as right or wrong, the reliabilities of the test scores were estimated as coefficient α (Cronbach, 1951) for each subpopulation under both treatments. The results are reported in Table 1.

Insert Table 1 about here

It is important to notice that the necessary statistics come from the correct experiment and not, for example, from ISS's in which students are already assigned to treatments on the basis of their scores on the placement test in question. In a proper experiment students from the same probability function of X are randomly drawn and assigned to treatments, after which their performances on the mastery test are measured.

As noted earlier, in each application of the combined decision problem it should be checked if the monotonicity conditions hold. Substituting the statistics of Table 1 into (13) and (15), showed that the set of conditions (16) was satisfied.

First, the quota-free situation is considered. Because the costs for testing are assumed to be equal for advanced and retained students, d_{h1m} is set equal to d_{h2m} $(h = 0,1)$. Similarly, the costs of following the different treatments j $(j = 0,1)$ are equal: $d_{j1p} = d_{j2p}$. Furthermore, it should hold that $b_{j1p} > b_{j2p}$ and $b_{h1m} > b_{h2m}$, taking into account the

fact that population 2 was considered more advantaged than 1. Using the computer program LINEAR, the optimal cutting scores x'_{ci} and y'_{ci} were then obtained by solving iteratively the system of Equations 19 and 20 for x_{ci} and y_{ci} (i=1,2) with t_c as starting values. The criterion for convergence was that the absolute differences between two iteration steps for both x'_{ci} and y'_{ci} were smaller than 10^{-7}. The results are reported in Table 2 for 3 different values of the utility parameters as well as for w = 0.5, 0.9, and 0.1 to illustrate the dependence of the results on the utility structure.

Insert Table 2 about here

The table shows that the consequence of raising the value of w is generally a decrease of the optimal placement scores and a small increase of the optimal mastery scores. Thus, increasing influence of the utility associated with the mastery decision implies that students should be assigned sooner to the "lower" treatment. In particular, the optimal placement cutting scores should be raised considerably for w = 0.1. This can be argued by the fact that the increasing influence of the utility associated with the mastery decision implies that students should be assigned sooner to the "lower" treatment in order to prepare them better for the mastery test at the end of the treatment. Besides, this better preparation for the mastery test accounts for the fact

that the optimal mastery scores can generally be set slightly lower with increasing w. Furthermore, inspection of Table 2 shows that both the optimal placement and mastery scores are lower for the advantaged than for the disadvantaged group. This is so because the disadvantaged students should be assigned sooner to the "lower" treatment. Also, they should stay longer in the instructional treatment to be sure that they have mastered the educational objectives.

In Table 2 the optimal cutting scores for the separate decisions are also displayed. The cutting scores optimizing the separate decisions were computed using Equations 27 and 31. As can be seen from Table 2, the optimal cutting points for the separate model do generally not have large differences compared to those in the combined model for w=0.5 and w=0.9 for both subpopulations. However, for w=0.1 the optimal cutting points for the placement decision of the combined model are substantially higher for both subpopulations. This can be explained by realizing that, as noted before, the psychometric portion of the separate model for optimizing the separate cutting scores does not depend upon the value of w. Furthermore, it has been argued before why the optimal cutting points for the placement decision of the combined model should be set rather high for w=0.1.

In the Introduction, it was remarked that one of the main advantages of the simultaneous approach was the increase of the overall expected utility. This can be demonstrated by comparing the gain in overall expected utility of the simultaneous to the separate approach. In order to calculate

the overall expected utility for the simultaneous approach, first the computed optimal simultaneous cutting scores from Table 2 were substituted into (9) for both subpopulations and the overall expected utility was calculated according to (10). The fourth term in the right-hand side of (9) has been computed using numerical integration methods, while the first three terms have been integrated analytically yielding respectively

$$(wb_{0ip}+(1-w)b_{0im})(t_c-\alpha_{0i}-\beta_{0i}\mu_{Xi}-\tau_{0i}\mu_{Y0i}) + wd_{0ip}+(1-w)d_{0im},$$

$$\{(wb_{1ip}-(1-w)b_{0i})(\theta_{1i}+\Gamma_{1i}\mu_{Xi}-t_c)+(wb_{0ip}+(1-w)b_{0im})(\theta_{0i}+\Gamma_{0i}\mu_{Xi}-t_c)+w(d_{1ip}-d_{0ip})\}\{1-\Phi[x_{Nci}]\} + \sigma_{Xi}\varphi[x_{Nci}]\{w(\Gamma_{1i}b_{1ip}+\Gamma_{0i}b_{0ip})+(1-w)b_{0im}(\Gamma_{0i}-\Gamma_{1i})\},$$

$$(1-w)\{\{(b_{0im}+b_{1im})(\mu_{Y0i}-t_c)+d_{1im}-d_{0im}\}\{1-\Phi[y_{N0ci}]\} + \sigma_{Y0i}\rho_{Y0i}(b_{0im}+b_{1im})\varphi[y_{N0ci}]\}.$$

Similarly, the overall expected utility for the separate approach was calculated by substituting the optimal separate cutting scores from Table 2 into Equations 25 and 28 for both subpopulations followed by the summation according to (29). The first and second term in the right-hand sides of (25) and (28) have been integrated analytically yielding respectively

$$(1-w)(b_{0im}(t_c-\mu_{Y0i}) + d_{0im}),$$

$$(1-w)\{\{(b_{0im}+b_{1im})\mu_{Y0i}-t_c)+d_{1im}-d_{0im}\}(1-\Phi[y_{N0ci}]) + \sigma_{Y0i}\rho_{Y0i}(b_{0im}+b_{1im})\varphi[y_{N0ci}]\},$$

$$w\{b_{0ip}(t_c-\theta_{0i}-\Gamma_{0i}\mu_{xi}) + d_{0ip}\},$$

$$w\{\{(b_{1ip}(\theta_{1i}+\Gamma_{1i}\mu_{xi}-t_c) + b_{0ip}(\theta_{0i}+\Gamma_{0i}\mu_{xi}-t_c) + d_{1ip}-d_{0ip}\}(1-\Phi[x_{Nci}]) + \sigma_{xi}(\Gamma_{1i}b_{1ip}+\Gamma_{0i}b_{0ip})\varphi[x_{Nci}]\}.$$

Computer programs EU-SIM and EU-SEP have been written to calculate the overall expected utility for the simultaneous and separate approach, respectively. Table 2 summarizes the results.

As can be seen from Table 2, the gain in overall expected utility for this particular example and chosen utility structures (1)-(9) is not very much. Only for utility structure (9) with w=0.1 the gain is substantially. This can be argued by the fact that the utility associated with the mastery decision is dominating in this case. Now, due to the high optimal placement cutting scores for the combined model, most students will be assigned to the "lower" treatment implying that on the average they are better prepared for the end-of-mastery test. As a result, due to the high positive utility associated with the advance decision for this particular utility structure, the overall expected utility will be rather large.

Note that for both approaches the overall expected utility increases with decreasing w. This means that the utility associated with the separate mastery decision

contributes the most to the overall expected utility.

Finally, the solution for quota-restricted placement for the combined model is considered. The proportions p_i (i = 1,2) were estimated as n_i/n, where n and n_i stand for the total sample and the number of students from the sample in subpopulation i, respectively. Furthermore, the fixed proportion p_0 of the total student population that could be assigned to treatment 1 was arbitrarily set equal to 0.333. The optimal cutting scores were then computed using the computer program QUO-SIM; the results are summarized in Table 3. The optimal solutions of the unrestricted model were used as first approximations in the iterative procedure. As mentioned earlier, before optimizing the cutting scores for quota-restricted placement, it should be checked if condition 37 holds. Inserting the solutions of the quota-free model, it appeared that this condition was satisfied for all values of the utility parameters and all values of w.

Insert Table 3 about here

As can be seen from Table 3, the optimal placement scores x'_{c1} and x'_{c2} in the quota-restricted model have to be raised substantially for the disadvantaged as well as advantaged groups compared to those in the quota-free model, because fewer students can be assigned to treatment 1 in the restricted situation. On the other hand, the optimal mastery

scores y'_{c1} and y'_{c2} in the quota-restricted model can be set slightly lower for both the disadvantaged and advantaged groups compared to those in the unrestricted model. This can be explained by realizing that only the "33.33% best" students are assigned to the "higher" treatment 1 implying that the remaining 66.66% low and average students are assigned to the "lower" treatment 0 where they are provided with more examples are exercises. As a result, the average student will be prepared better for the mastery test; hence, the optimal mastery scores can be set slightly lower.

Furthermore, it follows from Table 3 that, analogous to the unrestricted situation, the optimal mastery scores are higher for the disadvantaged than for the advantaged group. Unlike the quota-free situation, however, the optimal placement scores are higher for the advantaged than for the disadvantaged group for $w=0.5$ and 0.9 implying that disadvantaged students are sooner assigned to treatment 1 than advantaged students in these cases. This can be argued by the fact that, since the number of vacant places available for the "higher" treatment is restricted, otherwise hardly no disadvantaged students should be assigned to the "higher" treatment. However, since the influence of the utility associated with the mastery decision for $w=0.1$ is rather high, the disadvantaged students should be assigned sooner to the "lower" treatment in that case.

Also, the optimal separate scores for the quota-restricted model have been computed again. The optimal separate mastery scores are, of course, the same as those

obtained by the simultaneous approach. The optimal separate placement scores can be found by computing iteratively the system of Equations 32 and 39. Checking for condition 37, it appeared again that this condition was fulfilled; the results are shown in Table 3.

Finally, the overall expected utilities were calculated for the quota-restricted model analogously to the quota-free model. The results are also reported in Table 3. Note that, the overall expected utilities for both approaches is lower in the quota-restricted model. This result is in accordance with the theory of constrained optimization (see e.g., Bertsekas, 1982).

Discussion

In this paper an approach to simultaneous decision making for combinations of elementary decisions was described. The approach was applied to the area of instructional decision making by combining two elementary decisions (viz. a placement and a mastery decision) into a simple ISS. It was indicated that the optimal placement cutting scores obtained by the simultaneous approach in some cases differed substantially from those obtained by the separate approach. In particular, if it was assumed that the influence of the utility function associated with the placement decision was small, it turned out that the cutting points for the placement decision yielded rather large differences. It was indicated how by simultaneous optimization of such sequences

of decisions, optimal routing decisions could be taken using the framework of Bayesian decision theory.

The solutions given in this paper only apply to treatment assignment problems followed by an end-of-mastery test. However, more complicated decision networks can be handled effectively within a decision-theoretic framework. Further examination of the "best" way to represent more complicated decision networks of combinations of elementary decisions seems a valuable line of research. By simultaneous optimization of such sequences of decisions, optimal decisions can be taken using the framework of Bayesian decision theory. Also, restrictions such as multivariate test data and criteria can be taken into account. Furthermore, the optimization methods can be readily generalized to more than two treatments by introducing a series of cutting scores on the placement test.

Two final remarks are appropriate. First, it should be noted that the monotonicity conditions 2 and 4 are less restrictive than they look at first sight. As noted earlier, condition 2 should only hold if $u_{10i}(t)$ is a strictly decreasing function of t; that is if $0 \leq w < b_{0im}/(b_{11p}+b_{0im})$ in case of the linear utility model. Besides, since condition 2 is a sufficient condition, even if condition 2 does not hold for $u_{10i}(t)$ is a strictly decreasing function of t, the integrand of the second term of (9) still may be a nondecreasing function of x. This may be the case whenever $E_0[u_{00i}(T)|x]$ is a more slowly decreasing function of x than $E_1[u_{10i}(T)|x]$. For the linear utility model this condition

boils down to $\frac{d}{dx} E_{0i}(T|x) > \frac{d}{dx} E_{1i}(T|x)$ or $\Gamma_{0i} > \Gamma_{1i}$. Similarly, the fourth term of (9) still may be a nondecreasing function in each of its arguments x and y whenever $[E_0[u_{1im}(t)-u_{0im}(t)|x,y]n_{0i}(y|x)]$ is a more slowly increasing function in each of its arguments x and y than $[E_1[u_{1im}(t)-u_{0im}(t)|x,y]n_{1i}(y|x)]$.

Second, the example given in this paper was used only to illustrate the models. However, it is recommended not to use such small samples as in the described experiment, because the parameters to be estimated can yield errors of estimation in that case. If so, they can propagate in computing the derived optimal decision rules. This does not mean, however, that small samples necessarily yield inaccurate results. This is because not only distributional parameters but utility parameters as well determine how errors of estimation propagate.

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References

Bertsekas, D.P. (1982). Constrained optimization and Lagrange multiplier methods. New York: Academic Press.

Chuang, D.T., Chen, J.J., & Novick, M.R. (1981). Theory and practice for the use of cut-scores for personnel decisions. Journal of Educational Statistics, 6, 129-152.

Cronbach, L.J. (1951). Coefficient alpha and the internal structure of tests. Psychometrika, 16, 297-334.

DeGroot, M.H. (1970). Optimal statistical decisions. New York: McGraw-Hill.

Ferguson, T.S. (1967). Mathematical statistics: A decision theoretic approach. New York: Academic Press.

Fishburn, P.C. (1982). The Foundations of Expected Utility. D. Reidel.

French, S. (1986). Decision Theory: An introduction to the mathematics of rationality. Chichester: Ellis Horwood Limited.

Gross, A.L., & Su, W.H. (1975). Defining a 'fair' or 'unbiased' selection model: A question of utilities. Journal of Applied Psychology, 60, 345-351.

Hambleton, R.K. & Novick, M.R. (1973). Toward an integration of theory and method for criterion-referenced tests. Journal of Educational Measurement, 10, 159-170.

Huynh, H. (1976). Statistical considerations of mastery scores. Psychometrika, 41, 65-79.

Huynh, H. (1977). Two simple cases of mastery scores based on the beta-binomial model. Psychometrika, 41, 65-78.

Johnson, N.L., & Kotz, S. (1970). Distributions in statistics: Continuous univariate distributions. Boston: Houghton Mifflin.

Karlin, S., & Rubin, H. (1956). Distributions possessing a monotone likelihood ratio. Journal of the American Statistical Association, 1, 637-643.

Keeney, D., & Raiffa, H. (1976). Decisions with multiple objectives: Preferences and value trade-offs. New York: John Wiley & Sons.

Krantz, D.H., Luce, R.D., Suppes, P. & Tversky, A. (1971). Foundations of Measurement. Academic Press.

Lehmann, E.L. (1959). Testing statistical hypotheses. New York: John Wiley & Sons.

Lindgren, B.W. (1976). Statistical theory (3rd Ed.). New York: Macmillan.

Lindley, D.V. (1976). A class of utility functions. The Annals of Statistics, 4, 1-10.

Lord, F.M., & Novick, M.R. (1968). Statistical theories of mental test scores. Reading: M.A.: Addison-Wesley.

Mellenbergh, G.J., & van der Linden, W.J. (1981). The linear utility model for optimal selection. Psychometrika, 46, 283-293.

Novick, M.R., & Petersen, N.S. (1976). Towards equalizing educational and employment opportunity. Journal of Educational Measurement, 13, 77-88.

Novick, M.R., & Lindley, D.V. (1978). The use of more realistic utility functions in educational applications. Journal of Educational Measurement, 15, 181-191.

Petersen, N.S. (1976). An expected utility model for 'optimal' selection. Journal of Educational Statistics, 1, 333-358.

Petersen, N.S., & Novick, M.R. (1976). An evaluation of some models for culture-fair selection. Journal of Educational Measurement, 13, 3-31.

van der Gaag, N.L. (1989). Mastery decisions at the end of Dutch Secondary Education. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, March 27-31.

van der Linden, W.J. (1980). Decision models for use with criterion-referenced tests. Applied Psychological Measurement, 4, 469-492.

van der Linden, W.J. (1981). Using aptitude measurements for the optimal assignment of subjects to treatments with and without mastery scores. Psychometrika, 46, 257-274.

van der Linden, W.J. (1985). Decision theory in educational research and testing. In T. Husen & T.N. Postlethwaite (Eds.), International encyclopedia of education: Research and studies. Oxford: Pergamon Press.

van der Linden, W.J. (1987). The use of test scores for classification decisions with threshold utility. Journal of Educational Statistics, 12, 62-75.

van der Linden, W.J. (1988). Applications of decision theory to test-based decision-making. In R.K. Hambleton & J.N. Zaal (Eds.), New developments in testing: Theory and applications. Amsterdam, The Netherlands: North-Holland.

van der Linden, W.J., & Mellenbergh, G.J. (1977). Optimal cutting scores using a linear loss function. Applied Psychological Measurement, 1, 593-599.

Vos, H.J. (1988). The use of decision theory in the Minnesota Adaptive Instructional System. Journal of Computer-Based Instruction, 15, 65-71.

Vos, H.J. (1990). Simultaneous optimization of decisions using a linear utility function. Journal of Educational Statistics, to appear.

Vos, H.J., & van der Linden, W.J. (1987). Designing optimal rules for instructional decision making in CAI systems. In J. Moonen & T. Plomp (Eds.), Developments in educational software and courseware. Oxford: Pergamon Press.

Table 1

Statistics Placement and Mastery Tests (X and Y)

Statistic	Disadvantaged			Advantaged		
	X	Y		X	Y	
		Treatment			Treatment	
		0	1		0	1
Mean	55.965	63.266	62.148	59.832	66.125	67.233
Standard Deviation	10.821	10.144	11.245	11.674	10.517	12.523
Reliability		0.764	0.813		0.744	0.791
Correlation		ρ_{01}=0.786	ρ_{11}=0.819		ρ_{02}=0.725	ρ_{12}=0.771

Table 3

Optimal Cutting Scores Quota-Free Placement with Linear Utility

No.	Utility Specifications Disadv.		Adv.		w	Cutting Scores (x'_{c1}, y'_{c1}) Simultaneous Disadv.	Simultaneous Adv.	Separate Disadv.	Separate Adv.	Overall Expected Utility Simultaneous	Separate
(1)	b_{01p}=3.5	d_{01p}=-3	b_{02p}=3	d_{02p}=-2	0.5	x'_{c1}=46.38	x'_{c2}=43.81	x'_{c1}=46.10	x'_{c2}=43.82	21.09	20.87
	b_{11p}=2	d_{11p}=-3	b_{12p}=1	d_{12p}=-3		y'_{c1}=53.22	y'_{c2}=51.84	y'_{c1}=52.63	y'_{c2}=51.47		
	b_{01m}=3	d_{01m}=-4	b_{02m}=2	d_{12m}=-4							
	b_{11m}=4	d_{11m}=-5	b_{12m}=2.5	d_{12m}=-5							
(2)					0.9	x'_{c1}=46.12	x'_{c2}=43.82	x'_{c1}=46.10	x'_{c2}=43.82	14.92	14.87
						y'_{c1}=53.25	y'_{c2}=51.85	y'_{c1}=52.63	y'_{c2}=51.47		
(3)					0.1	x'_{c1}=55.19	x'_{c2}=45.30	x'_{c1}=46.10	x'_{c2}=43.82	27.43	26.87
						y'_{c1}=52.58	y'_{c2}=51.77	y'_{c1}=52.63	y'_{c2}=51.47		
(4)	b_{01p}=3.5	d_{01p}=0	b_{02p}=3	d_{02p}=0	0.5	x'_{c1}=46.14	x'_{c2}=43.46	x'_{c1}=45.87	x'_{c2}=43.46	24.94	24.72
	b_{11p}=2	d_{11p}=0	b_{12p}=1	d_{12p}=0		y'_{c1}=53.04	y'_{c2}=51.56	y'_{c1}=52.45	y'_{c2}=51.17		
	b_{01m}=3	d_{01m}=0	b_{02m}=2	d_{02m}=0							
	b_{11m}=4	d_{11m}=0	b_{12m}=2.5	d_{12m}=0							
(5)					0.9	x'_{c1}=45.89	x'_{c2}=43.46	x'_{c1}=45.87	x'_{c2}=43.46	17.96	17.92
						y'_{c1}=53.08	y'_{c2}=51.56	y'_{c1}=52.45	y'_{c2}=51.17		
(6)					0.1	x'_{c1}=55.09	x'_{c2}=45.54	x'_{c1}=45.87	x'_{c2}=43.46	32.06	31.53
						y'_{c1}=52.39	y'_{c2}=51.44	y'_{c1}=52.45	y'_{c2}=51.17		
(7)	b_{01p}=5	d_{01p}=-2	b_{02p}=3.5	d_{02p}=-2	0.5	x'_{c1}=46.87	x'_{c2}=43.30	x'_{c1}=46.06	x'_{c2}=43.99	51.59	50.72
	b_{11p}=3	d_{11p}=-3	b_{12p}=2	d_{12p}=-3		y'_{c1}=53.06	y'_{c2}=51.63	y'_{c1}=52.53	y'_{c2}=51.28		
	b_{01m}=6	d_{01m}=-4	b_{02m}=5	d_{02m}=-4							
	b_{11m}=9	d_{11m}=-5	b_{12m}=7	d_{12m}=-5							
(8)					0.9	x'_{c1}=46.12	x'_{c2}=44.02	x'_{c1}=46.06	x'_{c2}=43.99	29.78	29.60
						y'_{c1}=53.15	y'_{c2}=51.65	y'_{c1}=52.53	y'_{c2}=51.28		
(9)					0.1	x'_{c1}=57.69	x'_{c2}=49.04	x'_{c1}=46.06	x'_{c2}=43.99	74.05	71.84
						y'_{c1}=52.49	y'_{c2}=51.37	y'_{c1}=52.53	y'_{c2}=51.28		

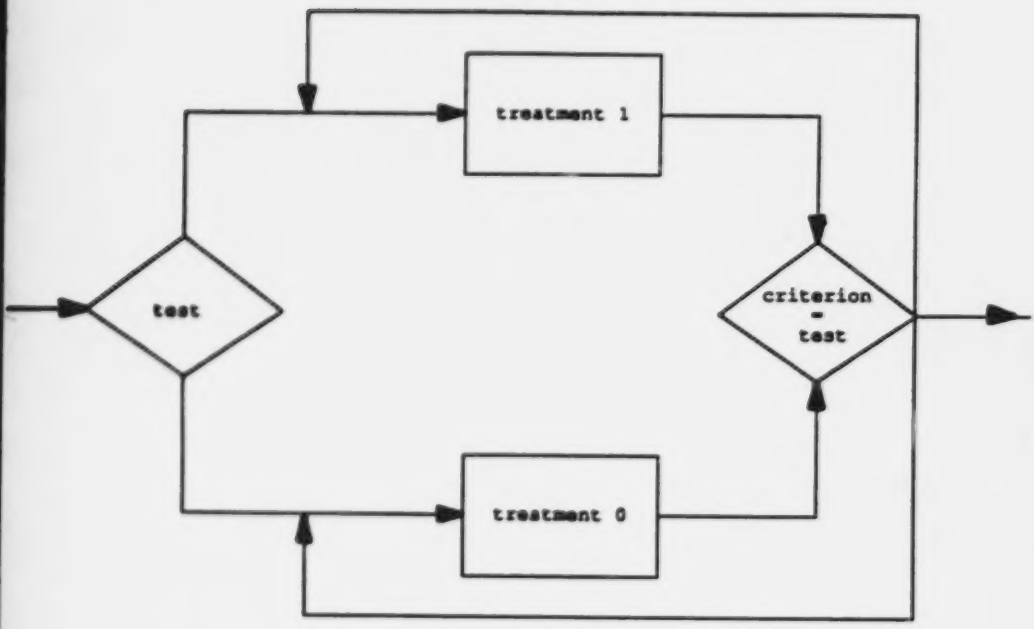
Table 3

Optimal Cutting Scores Quota-Restricted Placement with Linear Utility

No.	Utility Specifications				w	Cutting Scores (x^c_{t1}, y^c_{t1})				Overall Expected Utility	
	Disadv.		Adv.			Simultaneous		Separate		Simultaneous	Separate
						Disadv.	Adv.	Disadv.	Adv.		
(1)	b_{01p}=3.5	d_{01p}=-2	b_{02p}=3	d_{02p}=-2	0.5	x^c_{t1}=60.85	x^c_{t2}=64.04	x^c_{t1}=59.97	x^c_{t2}=65.11	12.62	12.19
	b_{11p}=2	d_{11p}=-3	b_{12p}=1	d_{12p}=-3		y^c_{t1}=52.62	y^c_{t2}=51.51	y^c_{t1}=52.63	y^c_{t2}=51.47		
	b_{01m}=3	d_{01m}=-4	b_{02m}=2	d_{12m}=-4							
	b_{11m}=4	d_{11m}=-5	b_{12m}=2.5	d_{12m}=-5							
(2)					0.9	x^c_{t1}=60.06	x^c_{t2}=64.97	x^c_{t1}=59.97	x^c_{t2}=65.11	-0.65	-0.76
						y^c_{t1}=52.62	y^c_{t2}=51.50	y^c_{t1}=52.63	y^c_{t2}=51.47		
(3)					0.1	x^c_{t1}=64.57	x^c_{t2}=60.53	x^c_{t1}=59.97	x^c_{t2}=65.11	26.05	25.13
						y^c_{t1}=52.64	y^c_{t2}=51.50	y^c_{t1}=52.63	y^c_{t2}=51.47		
(4)	b_{01p}=3.5	d_{01p}=0	b_{02p}=3	d_{02p}=0	0.5	x^c_{t1}=60.85	x^c_{t2}=64.04	x^c_{t1}=59.97	x^c_{t2}=65.11	16.22	15.79
	b_{11p}=2	d_{11p}=0	b_{12p}=1	d_{12p}=0		y^c_{t1}=52.44	y^c_{t2}=51.20	y^c_{t1}=52.45	y^c_{t2}=51.17		
	b_{01m}=3	d_{01m}=0	b_{02m}=2	d_{02m}=0							
	b_{11m}=4	d_{11m}=0	b_{12m}=2.5	d_{12m}=0							
(5)					0.9	x^c_{t1}=60.06	x^c_{t2}=64.97	x^c_{t1}=59.97	x^c_{t2}=65.11	1.84	1.83
						y^c_{t1}=52.43	y^c_{t2}=51.20	y^c_{t1}=52.45	y^c_{t2}=51.17		
(6)					0.1	x^c_{t1}=64.57	x^c_{t2}=60.53	x^c_{t1}=59.97	x^c_{t2}=65.11	30.65	29.74
						y^c_{t1}=52.45	y^c_{t2}=51.20	y^c_{t1}=52.45	y^c_{t2}=51.17		
(7)	b_{01p}=5	d_{01p}=-2	b_{02p}=3.5	d_{02p}=-2	0.5	x^c_{t1}=61.21	x^c_{t2}=63.65	x^c_{t1}=59.66	x^c_{t2}=65.50	39.60	38.34
	b_{11p}=3	d_{11p}=-3	b_{12p}=2	d_{12p}=-3		y^c_{t1}=52.53	y^c_{t2}=51.32	y^c_{t1}=52.53	y^c_{t2}=51.28		
	b_{01m}=6	d_{01m}=-4	b_{02m}=5	d_{02m}=-4							
	b_{11m}=9	d_{11m}=-5	b_{12m}=7	d_{12m}=-5							
(8)					0.9	x^c_{t1}=59.85	x^c_{t2}=65.25	x^c_{t1}=59.66	x^c_{t2}=65.50	7.55	7.14
						y^c_{t1}=52.51	y^c_{t2}=51.31	y^c_{t1}=52.53	y^c_{t2}=51.28		
(9)					0.1	x^c_{t1}=65.99	x^c_{t2}=59.49	x^c_{t1}=59.66	x^c_{t2}=65.50	72.14	69.37
						y^c_{t1}=52.54	y^c_{t2}=51.31	y^c_{t1}=52.53	y^c_{t2}=51.28		

Figure Caption

Figure 1. A system of one placement and one mastery decision


treatment 1
test
criterion
-
test
treatment 0

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